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The Hardy Cross method and its implementation in Spain

El método Hardy Cross y su implementación en España

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We do not need more than a piece of paper and a pencil to carry out the analysis of any reticular structure.

Carlos Fernández Casado

Abstract

In May 1930, Hardy Cross (1885-1959) published an article called 'Analysis of continuous frames by distributing fixed-end moments' in the American Society of Civil Engineers (ASCE). This article proposed a new approach to Structural Theory, and its relevance could be compared to that of the Three Moments Theorem (also known as the Clapeyron Theorem). The Cross method, as this calculation methodology has been often called, had remarkable significance from the moment it came out until the 70s, when new calculation methods became popular.

In the present article, we will be trying to evaluate its impact in locations far from its origins; in particular, how it was understood and formulated in Spain. As will be demonstrated, the importance of this method was extremely relevant for the construction of new buildings and the implementation of new industries, which started to appear in a decisive moment for the development of the country. Even though the Hardy Cross method was the most widely used methodology at the time, two other procedures were also available; namely, the Kani and the Takabeya methods, methods that would also appear in the technical bibliography of the time. Despite the infrequent implementation of these other methods, we have briefly referred to both of them in the present paper. *This article aims to show the relevance of the Cross method as well as its early implementation in Spain, by using academic bibliography of that time.*

Keywords: Cross method; Kani method; Takabeya method; Structural analysis; Iterative methods; Hardy Cross; Gaspar Kani; Carlos Fernández Casado.

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Resumen

En mayo de 1930, Hardy Cross (1885-1959) publicó un artículo titulado "Análisis de Marcos Continuos Mediante la Distribución de Momentos de Fin Fijo" en la Sociedad Americana de Ingenieros Civiles (ASCE). Este artículo propuso un nuevo enfoque de la teoría estructural, y su relevancia podría compararse con la del Teorema De Los Tres Momentos (también conocido como el Teorema de Clapeyron). El método Cross, como se ha llamado a menudo esta metodología de cálculo, tuvo notable importancia desde el momento en que salió hasta los años 70, cuando los nuevos métodos de cálculo se hicieron populares. Este método, así como muchas de aplicaciones adicionales, ha generado un gran número de artículos. En el presente artículo, intentaremos evaluar su impacto en lugares alejados de sus orígenes; en particular, cómo se entendió y formuló en España. Como se demostrará, la importancia de este método era extremadamente relevante para la construcción de nuevos edificios y la implementación de nuevas industrias, que comenzaron a aparecer en un momento decisivo para el desarrollo del país. Aunque el método de Cross era la metodología más utilizada en ese momento, también se disponía de otros dos procedimientos a saber, los métodos Kani y Takabeya, métodos que también aparecerían en la bibliografía técnica de la época. A pesar de la infrecuente aplicación de estos otros métodos, nos hemos referido brevemente a ambos en el presente documento. Este artículo tiene como objetivo mostrar la relevancia del método Cross, así como su pronta implementación en España, utilizando la bibliografía docente de la época.

Palabras clave: Método Cross; Método Kani; Método Takabeya; Análisis estructural; Métodos repetitivos; Hardy Cross; Gaspar Kani; Carlos Fernández Casado.

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1. INTRODUCTION

The Hardy Cross method has a clear aim, which is specified in the abstract of its first publication.

The purpose of this paper is to explain briefly a method which has been found useful in analyzing frames which are statistically indeterminate. The essential idea which the writer wishes to present involves no mathematical relations except the simplest arithmetic [1].

It is relevant to point out that at the time when the article was published (Fig. 1), the most common calculation methods were the Three Moment Theorem and the Slope-Deflection method. The first had been proposed by *Benoît Clapeyron* (1799-1864) [2], and the second had been created by Wilbur M. Wilson and George Alfred Maney [3], from the University of Illinois (1915). It is important to add, however, that one year before, Axel Bendixen had published the *alpha equations method for the resolution of portal structures* [4]. This proposed a similar formulation to the Wilson and Maney approach by focusing on the importance of the rotation and the displacements when calculating the state of a certain structure. These methods were simple in terms of their application but presented the difficulty of having to solve a system of n equations and n unknowns (depending on the type of structure). Therefore, the proposition of a faster calculation method was an extremely attractive prospect for structural analysts.

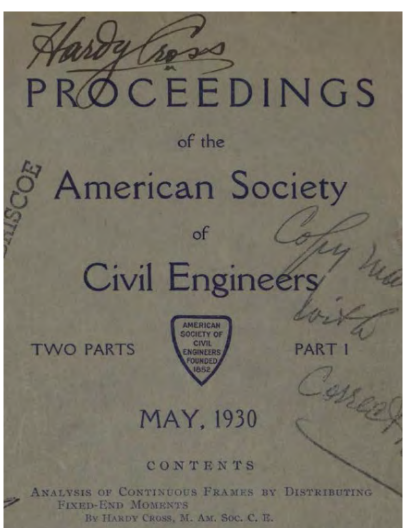

Fig. 1 American Society of Civil Engineers [1]

As a result of the proposal of the Cross method, the world of structural analysis changed dramatically. This change happened in the US, but it is also important to highlight the proliferation of so-called iterative methods in other countries. These alternative methods are presented in the following table, as well as further data on their place of origin and time when they appeared:

Table 1. The Cross, Kani and Takabeya methods, ordered according to relevance (in terms of bibliography)

Method	Cross	Kani	Takabeya
Owed to	Hardy Cross (1885-1959)	Gaspar Kani (1910-1968)	Fukuhei Takabeya (1893-1975)
Year	1932	1949	1938
Country	US	Serbia	Japan

From the moment of publication onward, the bibliography on the subject of the Cross method started to increase rapidly, and so did correspondent translations to multiple languages [5]. However, it is important to state that in the present article we will be focusing on the method's impact in Spain, regardless of its huge relevance all around the world.

The Cross method in Spain

Its application started in the 30s, due to Carlos Fernández Casado (1905-1988), a remarkable civil engineer [6]. In reference to the North American's method he said: "we immediately adopted that calculation method for our projects and it appeared for the first time in a public tender in 1932, where we used a load simplification and symmetry on structures". It is relevant to state that Carlos Fernández Casado had already heard of the method since the time of its publication in 1930. From that moment on, its application had become increasingly more popular. It is well known that Spain suffered a tremendous civil war during the period 1936-1939 [7]. This situation, as well as its extremely long post-war aftermath, resulted in significant drawback for the economic development of the country. Eventually, the Spanish government set what was called *Plan de Estabilización* [8] (Plan of Stabilization), which was aimed at increasing economic activity. Indeed, this plan introduced several measures which helped the country start to recover economically. This plan explains the significant increase in the creation of

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new infrastructures, new buildings and new factories to stimulate this economic recovery. This growing economy led to a rush in new projects. The calculation methods that had been used until then soon became obsolete, so there was a need to either find new methodologies, or to reconsider alternative calculation methods, which was the case of the Cross method.

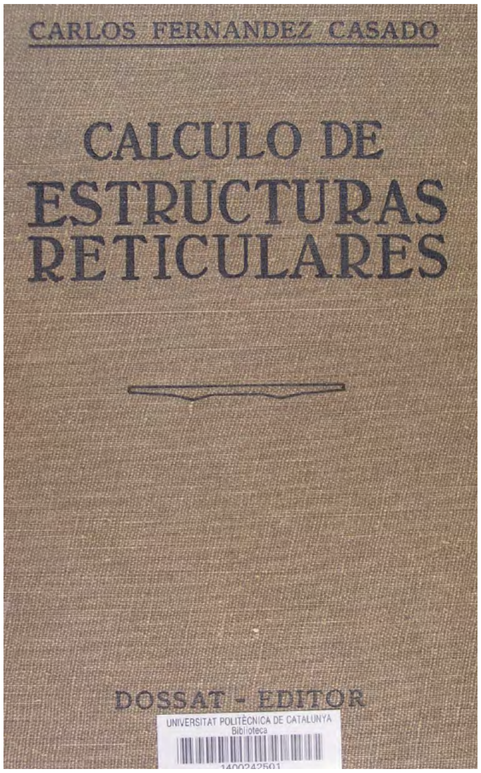

Fig. 2 *Cálculo de Estructuras Reticulares. Nudos* ***rígidos***. Carlos Fernández Casado [9].

As a result, both technical engineers and students needed to learn the method in order to be able to apply it correctly. Carlos Fernández Casado, a very young but recognized engineer, was one of the most important disseminators of the method at the national level. Soon after, his book *Cálculo de Estructuras Reticulares. Nudos Rígidos* (Fig. 2) (*Resolution of reticular structures. Rigid Nudes*) became a must-have in every engineer's library and was considered the seminal reference book for almost any engineering and architecture student at a Spanish university. In fact, its relevance shaped the knowledge of a whole generation of structural analysts.

Formulation of the Hardy Cross Method

Throughout the book, Fernández Casado highlighted the relevance of the Cross method, as it can be seen in the following passage.

We present a complete systematization of the Cross method. This method, considered by the Americans to be "the most valuable contribution to the structural analysis field during the first 30 years of the 20th century," solves in a much simpler and accurate way any kind of reticular structure, regardless of how complex it might be. There are calculation methods in which direct intuitions do not only focus on the point of start but also have an influence on the whole process, by giving to the calculation transformations a more valuable sense than the merely combinatory. The common points between the physical and mathematical realities are constant all along the process, and through abstract operations the reality of the physical phenomenon arises. It is to these kinds of methods that our adaptation of the Cross method refers [9].

In addition,

Practical advantages: by having the intuition of the physical phenomenon, which has to be developed through the Structural Theory, nothing more is required; the problems are solved without need of having to remember ingenious combinations, therefore in a simpler, safer and more pleasant way [9].

The dissemination of the method was extremely fast, and almost in every report on structural analysis there were examples fully or partially calculated through the Cross method (Figs 3-4).

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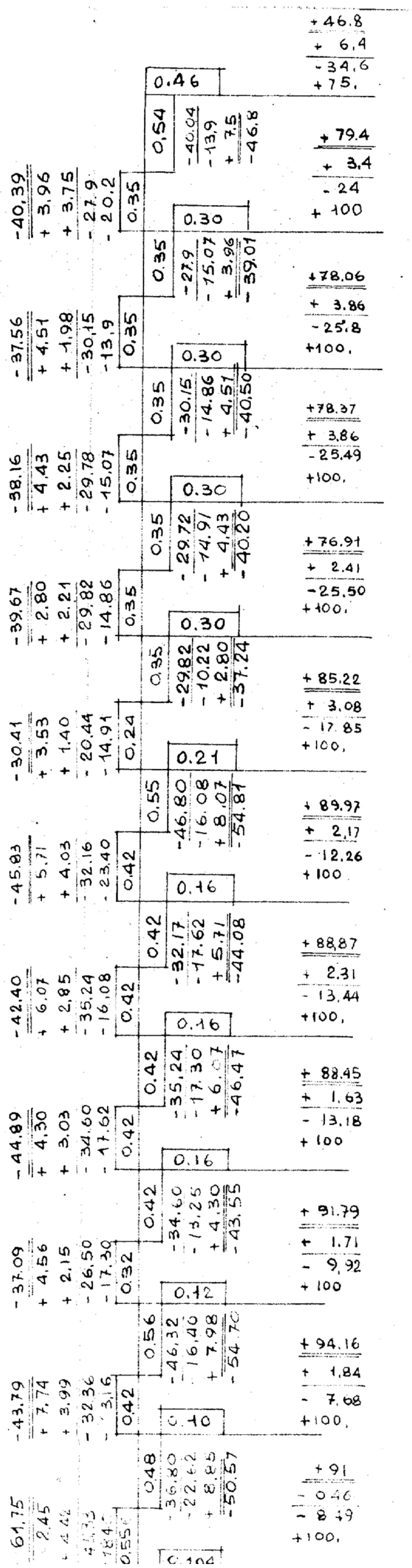

Fig. 3 Example of the Cross method (I). Barcelona Superior Technical College of Industrial Engineers [10].

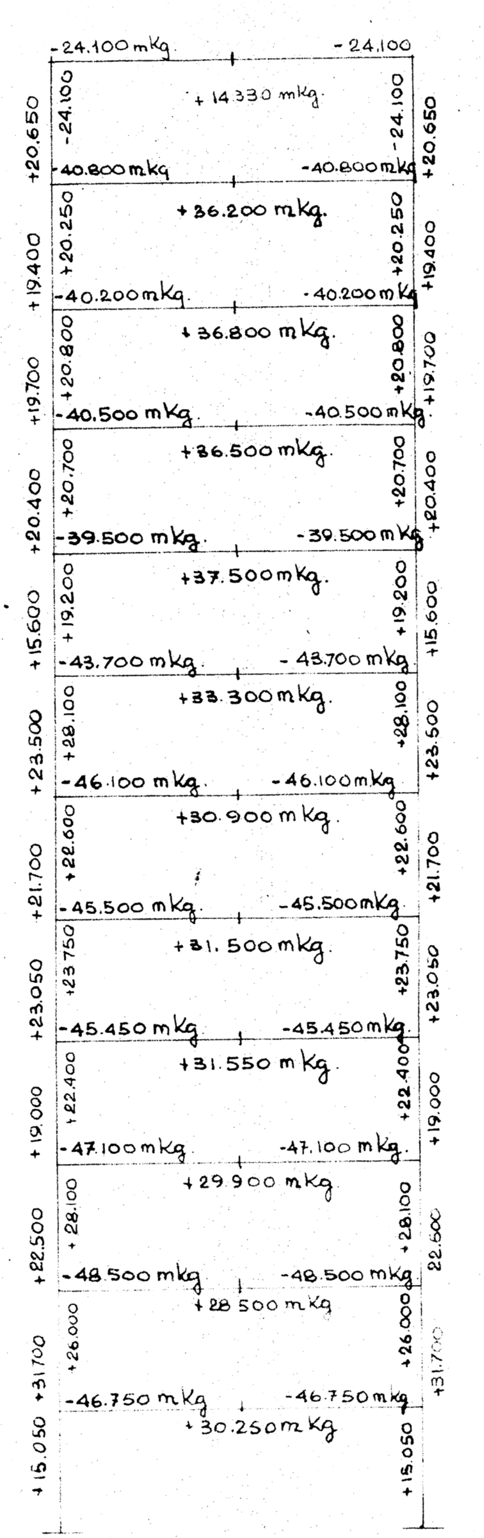

Fig. 4 Example of the Cross method (II). Barcelona Superior Technical College of Industrial Engineers [10]

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Fig. 5 Photo of the building during the construction (left) and at present time (right). Fuente: Barcelona Superior Technical College of Industrial Engineers.

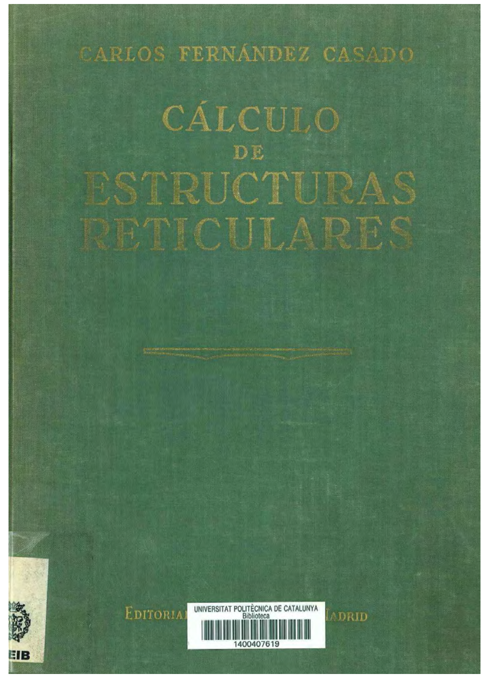

Fig. 6 *Cálculo de Estructuras Reticulares. Nudos rígidos*. Carlos Fernández Casado [12].

Fernández Casado presents the method didactically and approaches it by showing the reader a wide variety of examples involving different geometries. Before setting the formulation of the method, he analyzes the 'bar' element. He states:

Provided that we are dealing with structures whose geometry is discontinuous and formed by bars, the method has to adapt to such decomposition, by using the 'bar' element as the reference unit and by understanding all the possible relations through this decomposition from the beginning [11].

Once the relations on an isolated beam are known, we can focus on the study of the structure, understood as a set of bars;

The problem of the 'bar' can be reduced to the problem of determining the moments at the extremes of the correspondent beam; therefore, solving the problem of the whole set of bars will consist on finding these two moments for each beam of the structure [13].

The difficulty of the calculations of a set of bars, as it can be clearly seen in other analytical methods that have been used lately [14], is on the resolution of the associated linear system of equations, which becomes highly complex when the number of equations is very high. The Cross method solves this difficulty [15].

Another aspect that had traditionally been considered to be difficult and had been often omitted was the obtainment of the displacements. The Cross method was helpful as well in terms of easing this process.

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Once these previous comments have been made, the author moves on to the formulation of the method. Again, he focuses on the informative goal of the article through a great number of illustrated examples to help both technical staff and students fully understand the procedure.

We start by considering a virtual structure with all nudes assumed completely rigid (in other words, these nudes cannot rotate nor have any displacements), and we will proceed to eliminate this additional stiffness by allowing rotations at first and allowing displacements in a further stage [17].

The steps that are considered are presented in the following table [17]:

- **First step.** Completely rigid nudes.
- **Second step.** Nudes allowed to rotate but no displacements.
- **Third step.** Displacements allowed on the nudes, but no rotation possible.
- **Fourth step.** Nudes allowed to both rotate and move.

Once the steps I and II have been carried out (these steps were named by the author as *fundamental stage*), the structure is solved in case it is non-translational; in other words, if there are no displacements on the structure (non-translational structure), the process is over and the moment diagram we have just obtained is the solution we are looking for. Once this moment diagram is obtained, it is very easy to obtain the rest of the diagrams, since the structure is isostatic.

We present right after an example of the Cross method (steps I and II), obtained from a calculation report from Escola Tècnica Superior d'Arquitectura de Barcelona (ETSAB) -Barcelona Superior Architecture College- (Figs. 7-8). This document was created in 1961-62 by the architect Eusebi Bona (1890-1972).

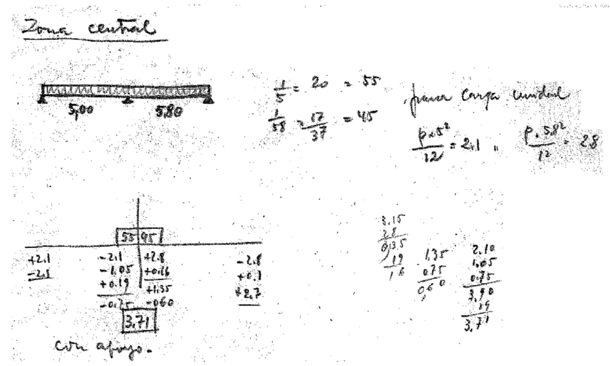

Fig. 7 Example of the Cross method. Fundamental stage [18]

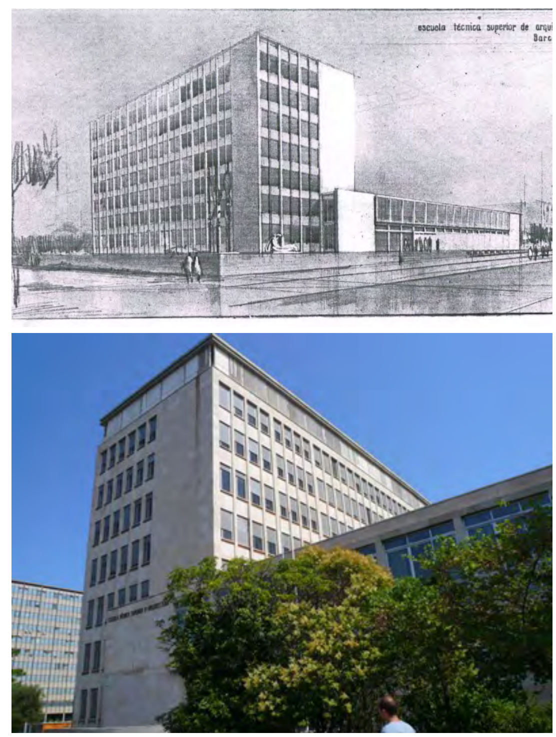

Fig. 8 Building during the design phase and at present time (right) [19].

Nevertheless, if the structure has horizontal displacements, commonly regarded as Δ, the structure is translational. This would be the case of non-symmetric structures, either with a non-symmetric geometry or non-symmetric loads. For these cases, we have to continue and apply the so-called *parametric stage*, which would correspond to the third and fourth steps of the method.

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Therefore, the final result is:

> The values for the total moments can be deduced from the superposition of the ones obtained in step II with the ones we have just obtained in step IV, which have been previously multiplied by the corresponding coefficient [20].

The author describes these steps graphically:

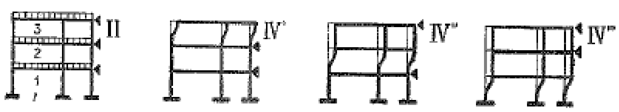

Fig. 9 Translational structure [15].

Further examples in the Kingdom of Spain

We wish to stress the pre-eminence of Hardy Cross's method in Spain, by way of some examples which showcase its influence on a number of structures solved using the Cross method.

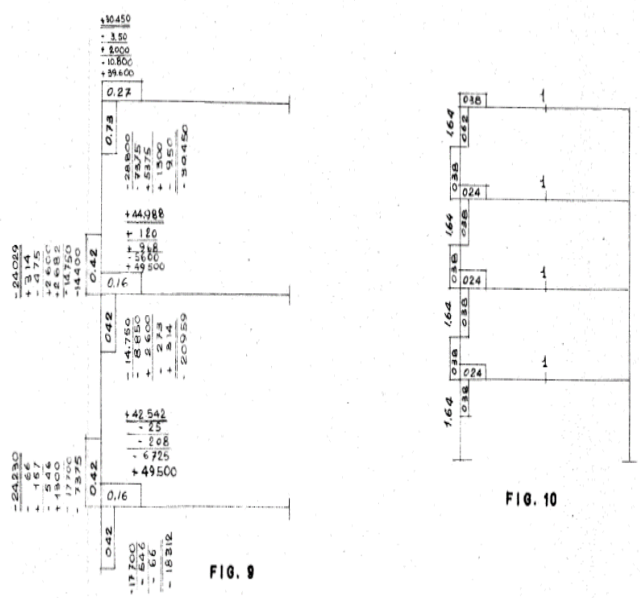

Fig. 10 Example of a porticoed structure calculation. Pavilions of the Barcelona Superior Technical College of Industrial Engineers [10].

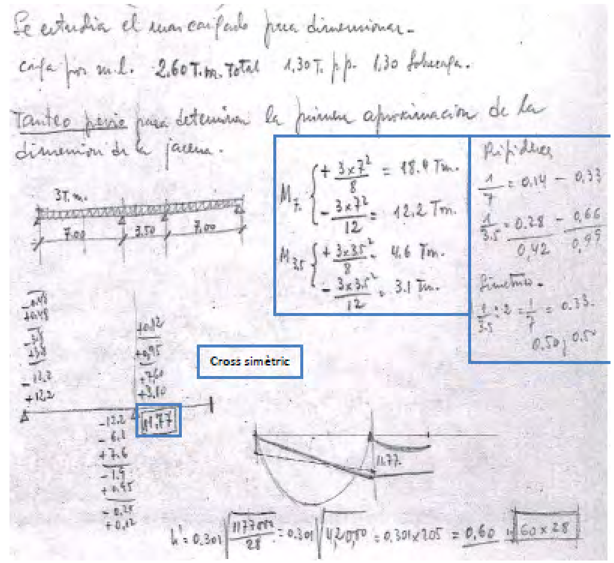

Fig. 11 Example of a continuous beam calculation. Barcelona Superior Architecture College [21].

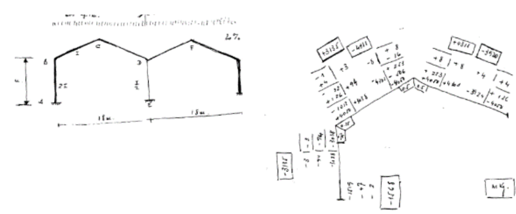

Fig. 12 Example of a gabled roof calculation [22].

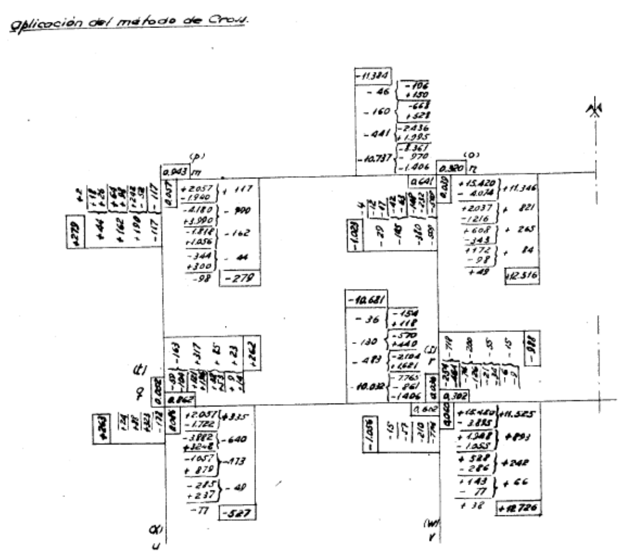

Fig. 13 Example of a concrete porticoed structure calculation [23].

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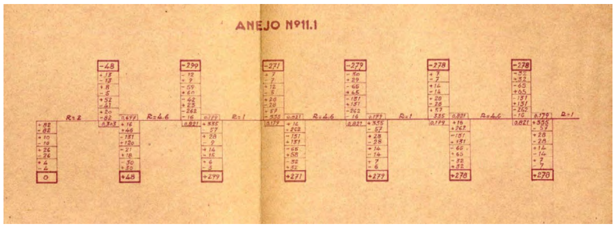

Fig. 14 Hangar calculation [24].

Our intention was to show the reader instances of differing typologies in order to reassert the widespread applicability of the method which rendered the solution of any hyperstatic structural typology possible.

The previous examples are but a mere sample of the importance the method had in Spain, given that references to said calculation procedure has been found both in consulted reports and in multiple bibliographical and teaching references from that specific time..

Other calculation methods

The authors have decided to refer to two alternative calculation methods, which despite being conceptually correct and being explained in the technical bibliography of the time, were significantly less popular than the Cross method. In fact, we can even say that, at the present time, these methods have been forgotten, at least in Spain. These methods are commonly referred as the Kani and Takabeya methods, and were created by Gaspar Kani and Fukuhei Takabeya, respectively.

The Kani method: a forgotten calculation method

The Hardy Cross method has been the most widely used method of its kind, and the one that has generated the broadest bibliography in Europe, which may explain why the other two methods mentioned have not been used as often. However, the other methods have been proved to be equally valid and to provide correct results. In the following section, we will present the Kani Method (created by Gaspar Kani [25]).

The method

In 1949, Gaspar Kani presented a calculation method which provided an *exact result based on successive approximations* [26]. The author tried to address the problem of finding the horizontal displacements of a structure (as a response to the Hardy Cross method). Kani looked for an easier way to calculate the horizontal displacements.

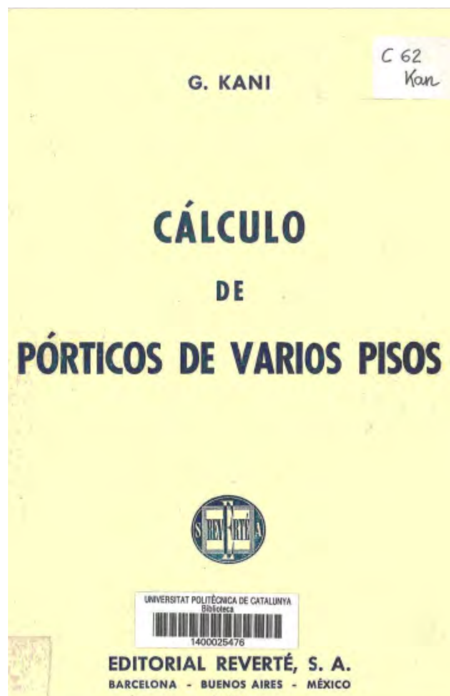

Fig. 15 *Cálculo de pórticos de varios pisos*, Spanish edition 1958 [27].

Carlos Fernández Casado referred to the method in the following way:

An interesting method within the group of the methods based on distributing moments, particularly suitable for structures that are composed by bars with constant inertia and are built as a grid [28].

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The bibliography associated with the method, when compared to the Hardy Cross method, is not abundant, as already noted. Indeed, this fact shows how the dissemination of this method was not as widespread. For instance, the book containing the Kani method was not edited in Spain until 1958, when the American method had already had a huge impact.

The advantages of the method with respect to other alternatives, the way Gaspar Kani saw it, are:

- By assuming rigid nudes (in other words, by not considering the possible displacements of the nudes), the calculations have a 'corrective' effect, so that, apart from saving time, this method allows the avoidance of mistakes.
- The consideration in the hypotheses that the nudes may have displacements induces a very small modification on the calculations, so it has a very limited impact.
- The process of checking the results can be made at every nude at any time, without the need of being aware of the calculations that have been done to reach the final result.
- In case the loads or the geometry of the bars were to be changed, the new solution would not have to be computed from scratch, since we will have to only compute again specific calculations [29].

Fundamentals of the method

The author introduces the method by starting from a hypothetical bar *i-k* which is under a generic distribution of loads, like the one shown in Figure 16.

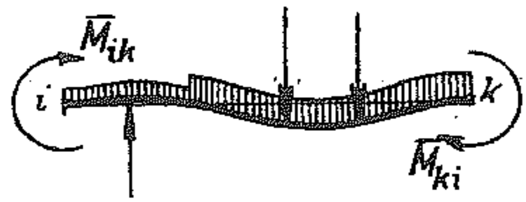

Fig. 16 Bar *i-k* under a generic distribution of loads [29].

Both the nudes *i* and *k* will rotate a certain angle. The value of such rotation can be decomposed as a superposition of three stages which will lead to finding the values of the moments at the extremes.

Stage I

Embedded nude; bending because of the loads: definition of embedded moments $\mathbf{M}_{ik}$.

In order to find these moments, the author recommends the following bibliography [29]:

- *Beton Kalender*.
- *Stahlbau Kalender*.
- *Takabeya Rahmentafeln*.

Stage II

The extreme i rotates a certain angle $\boldsymbol{\tau}_i$. *The k extreme does not rotate* (Fig. 17).

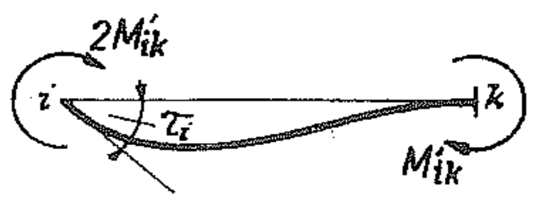

Fig. 17 The *i* extreme rotates an angle equal to τi [34].

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Stage III

The k extreme rotates a certain angle τ_k*. The i extreme does not rotate* (Fig. 18)

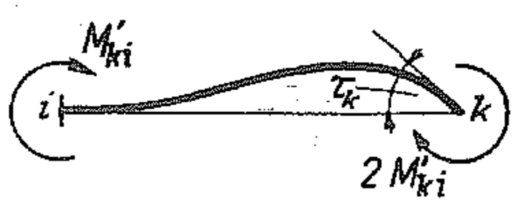

Fig. 18 The *i* extreme rotates an angle equal to τk [29].

Therefore, by applying the superposition principle, in the *i* extreme of the bar *i-k* the value for the resultant moment will be:

In other words, the total moment can be found as a sum of the embedded moment, the moment due to the rotation of the opposite extreme and the moment due to the rotation at that point multiplied by two.

We can conclude by setting out once again how this method is perfectly valid and how it can reach a considerable level of accuracy, as Gaspar Kani proposed in his work.

> An advantage of the method is that the errors committed along the computations are removed with the successive iterations. The probability of committing an error is very small since the method is based on repeating an extremely simple arithmetic operation where any misplacement of any sign is almost impossible. However, in case any error was made, this would not affect the global result, as long as this error has not been made on the subjection moments or the distributing coefficients.

We will consider the end of the process once the values we obtain are almost equal, and since the errors are highly improbable, we can assume this last value found from the iterative process is the correct value we are looking for [29].

The Takabeya method: another forgotten calculation method

On 1938, the Travaux Journal published an article named Étude *des ossatures de gratte-ciel composées de cadres rectangulaires et à joints rigides sous l'action du vent* [29], the article by Fukuhei Takabeya where a new calculation method was proposed. This is the methodology that would eventually be referred as the Takabeya method.

Fig. 19 Fukuhei Takabeya [30].

As it has been previously discussed, this is a method that had some importance in Spain (in terms of professional engineers being aware of the method). Still, it was not used at all in the field of structural calculations. According to the author:

In the project of modern constructions, the importance of the structural calculations increases every day. When a set of charges is applied to a rigid structure, its different elements undergo elastic deformations that are controlled by the rotations in its nudes and its own structural elements. In order to analyze the distribution of stresses in a structure, several methods exist, but they are difficult and tedious to use [31].

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It is especially interesting to point out how the author refers to the alternative methodologies from Hardy Cross and Gaspar Kani,

The Hardy Cross Method provides a practical solution to hyperstatic structures, but in case the nudes are not fixed (i.e., they can move when charges are applied), this method is not as useful as the one based on the deformation angles that is currently proposed. For instance, for a hundred-floor building where the force of the wind is not negligible and there are hundreds of deformation angles, both the Cross and Kani Methods are excessively long and tedious [31].

Therefore, he states:

The proposed method proved its usefulness when a problem of a 200-story structure was solved in 78 hours, which is an impressively good time score for such a complicated problem. This methodology was made available to the public in Paris in 1938 [31].

2. CONCLUSIONS

In the beginning of the article we pointed out the impact that the Cross method had in the structural analysis world [32][33][34]. This method enabled to solve structural problems extremely quickly regardless of the geometry [35]. The relevance of the method soon spread all over the world and arrived in Spain, where there was a very rapid dissemination of the method. In this article and with the help of the additional documentation we have referred to, we have tried to set out how the method was explained in the Spanish technical teaching institutions [35]. In addition, we have tried to give an overview on how this information was used in academic bibliography because this is a key factor to understand how a whole generation of technical professionals was formed.

Despite the great impact of the method, which can be seen through the large amount of calculation reports that used it, we must refer to other equally valid calculation procedures. These are the Kani and Takabeya methods, created by Gaspar Kani and Fukuhei Takabeya, respectively. This is why we have briefly presented the formulation of the Kani method. As it can be seen, it is an iterative method that does not deal with any type of mathematical complexity. Indeed, there are several aspects that are similar to the Hardy-Cross methodology, widely regarded as the great iterative method of the 20th century. However, despite the validity of the Kani method, it had a smaller impact than the American method.

Years after that, iterative methods were progressively replaced by numeric methods which were gradually appearing and which were to become the prevailing structural methods thanks to the general use of electronic and IT devices [36].

Nowadays, iterative methods (among them, the one named after Hardy Cross) have been largely relegated to technical colleges were the instruction of Structures Theory is essentially based on numeric methods, logically influencing their graduates who then go on to join the labor market.

Despite this, we believe it is important to acknowledge, as a conclusion, the influence that the method of the American structural engineer Hardy Cross has exercised, especially during the 20th century, on the education of generations of prospective scholars who have had the chance to apply it to their daily work. Proof of this widespread, enduring imprint are the manifold, and typologically varied structures which carry the legacy of the Hardy Cross method throughout the structural engineering world.

3. CONFLICT OF INTERESTS

The autor declare that there is no conflict of interest with an institution or commercial association of any kind.

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